EQUITY SYSTEM

OF

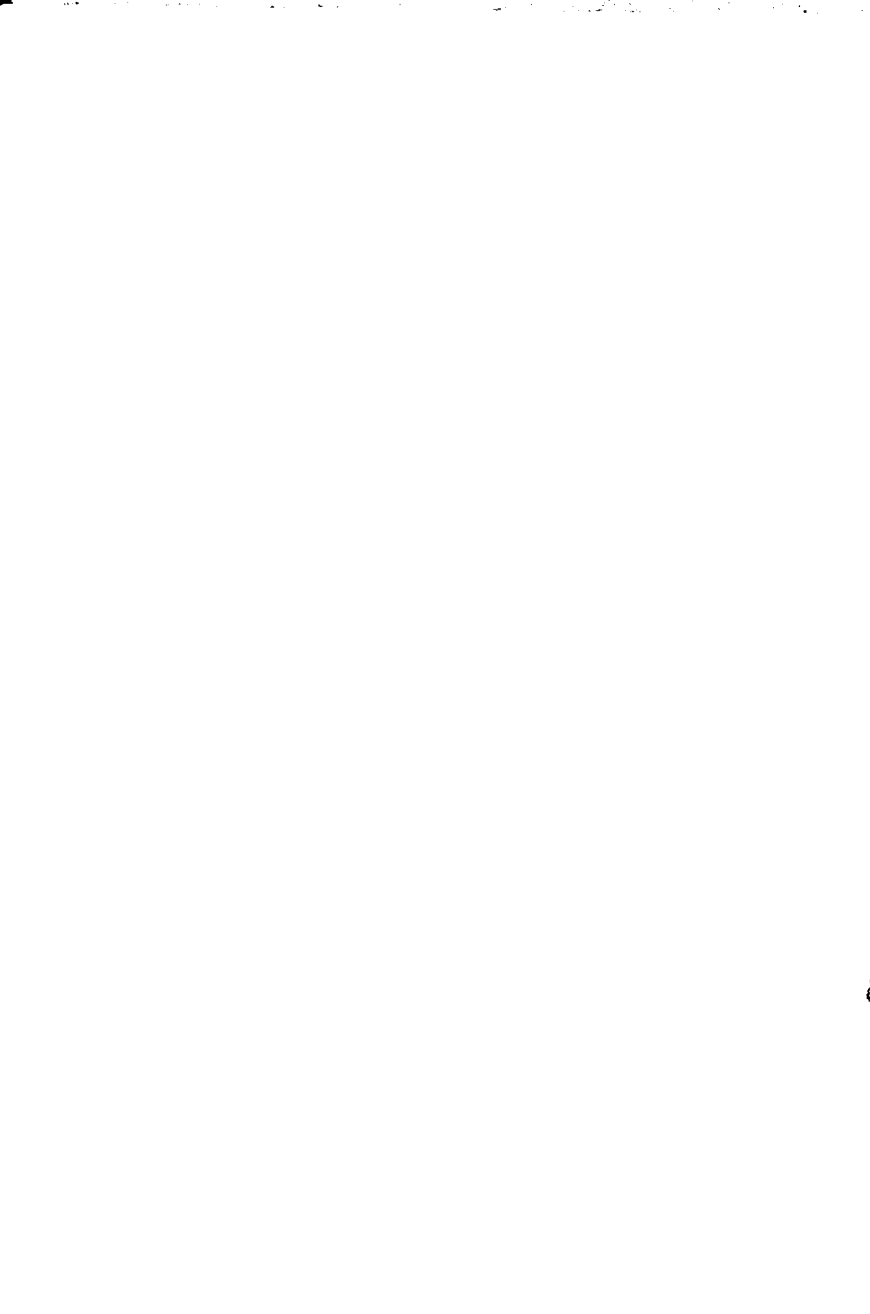# Cumulative Tontine Investment.

A SYSTEM OF EQUITABLY DISTRIBUTING THE PROFITS DEVELOPED

BY LIFE INSURANCE AND TONTINE PRINCIPLES AS APPLIED

TO THE

"BOND ENTERPRISE."

BY

D. H. SLEEM, B. A., M. D.

NEW YORK.

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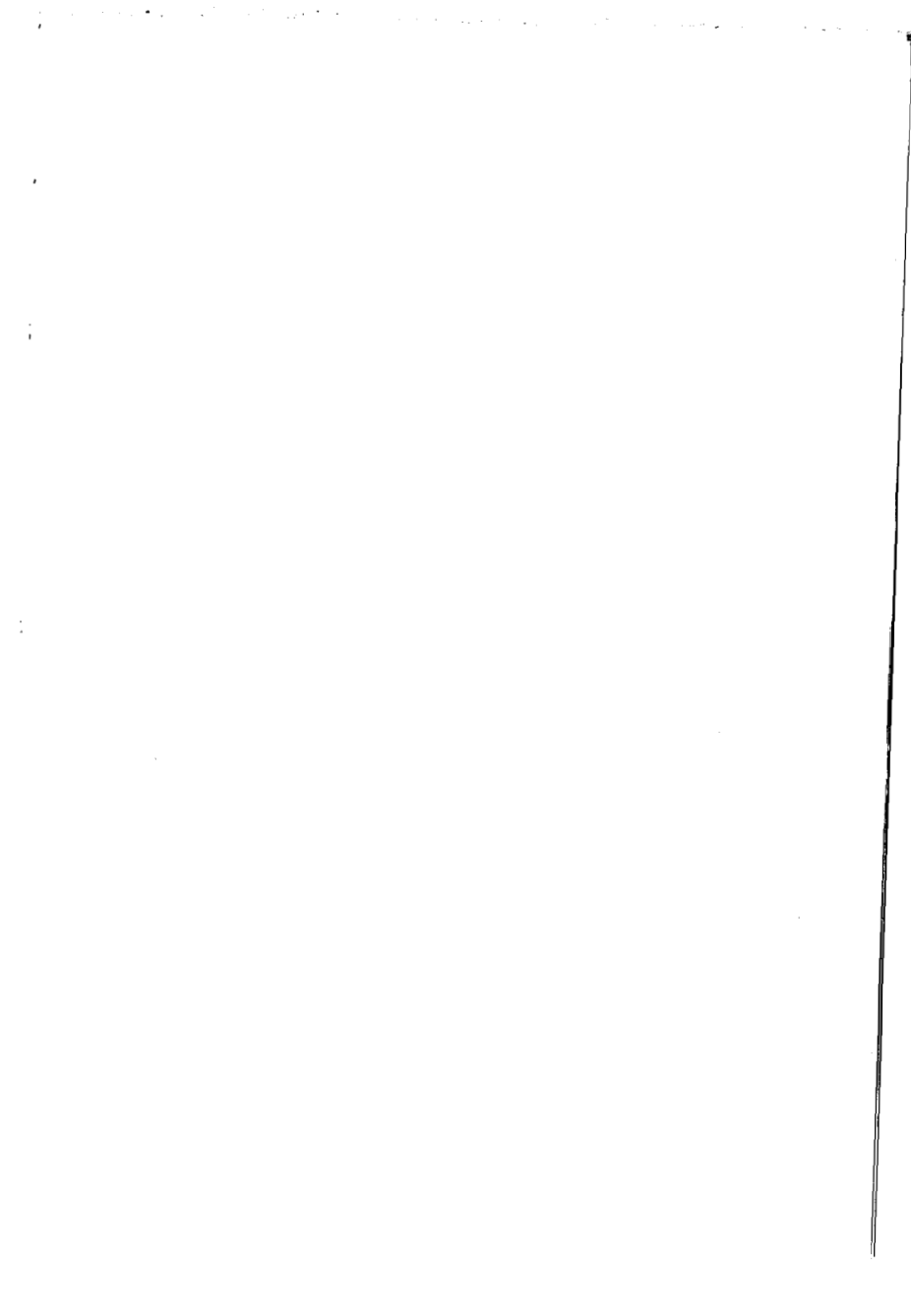BY LIFE INSURANCE AND TONTINE PRINCIPLES AS APPLIED
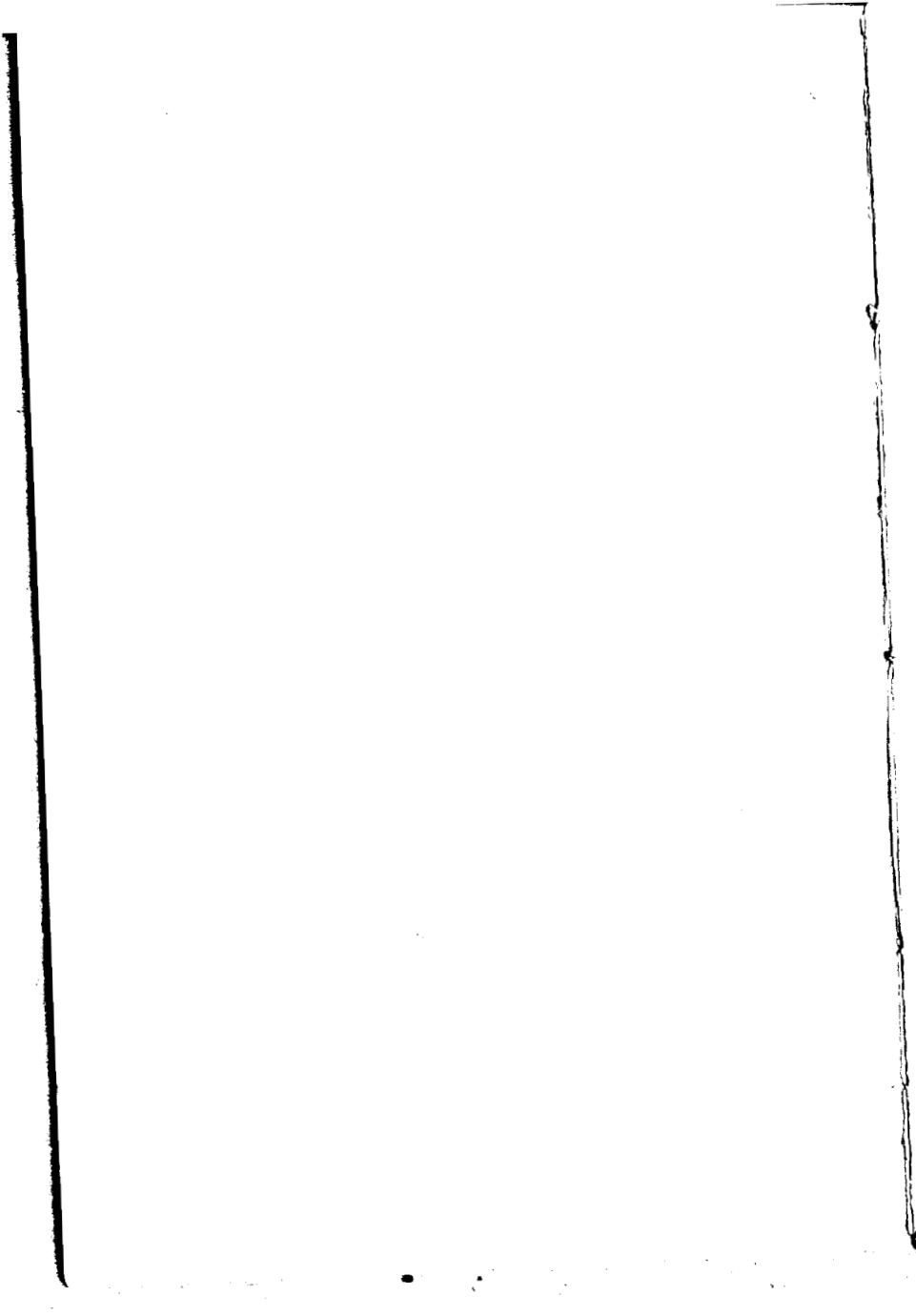
TO THE

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EQUITY SYSTEM

CUMULATIVE TONTINE INVESTMENT.

In order to do full justice to the elucidation of this system which the author will present in the following pages, allusion will be made to its proto-type, the Life Insurance system. But such allusion should not be construed as hostile nor condemna-tory to the practical principle upon which the Insur-ance system is based.

Ever since Lorenzo Tonti, the Italian banker, brought forth, in the seventeenth century, his new theory of co-operative investment, whereby the long liver is benefited by the death of his associates,—or, as applied in life insurance, that the persistent be benefited by the neglect of the non-persistent,—the financial world has been awakened to a new de-parture in finance, which to-day gives an impetus to some of the largest financial institutions in the world.

The reverse of Tonti's theory suggests the prac-ticability of the Insurance system, where it is the

one who dies early who obtains an advantage for his heirs at the expense of the long liver. Though his principle is the converse of the ordinary Life Insurance, yet it has been introduced into that system, for the distribution of so-called profits assigned to the long liver at the expense of those who forfeit their policies by their neglect to pay, in due time, the amount agreed upon in the contract.

The Tontine principle has been brought into considerable prominence by the leading insurance companies of this country and Europe.

Dr. John H. Durland, of New York, who has a vast experience in the statistics of life insurance, and who first compiled the tabulated results of the American Life Insurance Companies, published by "The Spectator" Co. of New York, covering the results of all business transacted before the organization of the State Insurance Department, was the first to conceive, in 1889, the happy idea of applying the same Tontine principle for the current benefit of the living instead of the heirs of the dead, as it is now practiced by the life insurance companies, by bringing before the public his new system of "Monthly Redemption Bonds," which system has come to stay, and has already obtained a prominent standing in the financial world.

It is not the aim of the writer to introduce at present, either a new principle, or a new application of the Tontine principle, but he seeks to provide a method by which the Tontine profits are distributed equitably among the persistents, as applied to the system of "Monthly Redemption Bonds."

PART I.

THEORY AND PRACTICE OF LIFE INSURANCE.

In theory, the Life Insurance system is based upon the computation of a sum of money received in installment by the insurance companies, and paid back, with compound interest, to the policy holder at the end of the expectation of life when the policy is presumably to terminate by the death of the holder. In practice such theory is inadequate to account for the enormous profits that the insurance companies have sometimes to pay, by virtue of their contracts, over the premiums they receive.

The following table, which is computed from life insurance rates, verifies the above assertion. It shows the amount of insurance that $100 would secure at age of 30, and also the amount of $100 with 5% compound interest.

Should one die at age of	His heirs would receive from the Insurance Company	his money with 5% comp. interest
30	$4291.84	$100.00
31	4291.84	205.00
32	4291.84	315.25
33	4291.84	431.01
34	4291.84	552.56
35	4291.84	680.19
36	4291.84	814.20
37	4291.84	954.91
38	4291.84	1102.66
39	4291.84	1257.79
40	4291.84	1420.68

In the second column, no account is taken of a probable dividend which may be added to $4291.84, and render that amount much greater.

While it may be said that the one at age of 30 is expected to live about 35 years, yet the probabilities are sometimes reversed, in spite of all precautionary measures; the weak may live to old age, and the strong may die in the years of his strength. The possibility of death at any age, renders the theory of compound interest less operative, and introduces into the Life Insurance system a speculative investment element.

By permission of the author I quote from a pamphlet entitled "Tontine Investment Bonds," by Dr. J. H. Durland, the following: "From thirty-six recent ·weekly statements issued by The Mutual Life of New York,' the Company reports $6,079,100 paid for claims upon which the insured paid $2,246,814. The Company therefore returned nearly $3 for one paid to them," an average of 200 per cent. Could the compound interest theory *alone* account for such enormous profits in face of the fact that any policy may terminate the next day after the first premium had been paid? How and in what manner this can be done in life insurance as well as in the "Monthly Redemption Bonds," the reader is referred to the above work for full information.

However, it might be said in this connection that, aside from the working of compound interest, the *lapse element* and the *current revenue* from new and old business, are the two important factors that enable the insurance companies to pay such large

profits and accumulate, besides, the hundreds of millions of dollars in their treasuries.

A STRIKING FEATURE

in the Life Insurance system, from a co-operative investment point of view, is the *unequal distribution of benefits.*

A policy of $1000 may be paid for the premium of $20 or $50 in one instance, while in another, the same amount may be paid for $600 or $800 *or more.* This seems to be unreasonable and contrary to the very principles of investment. For the profits in any legitimate enterprise should depend upon the amount invested as well as upon the term of investment.

Keeping this in view, a Tontine Policy or Bond should increase in redemption value, if it were to have any, as the amount paid therefor increases from one period to another, whether that period be a year or a month; the ratio of such increase should be the same to all policies or bonds of the same age and amount; and last, but not least, this ratio, by virtue of the Tontine principle, should be cumulative, for the longer a policy runs the more valuable it should become, because of the profits it accrues from the accumulation of the reserve in which it has an interest.

The ordinary life policy does not embody these features, and therefore cannot be called good investment in the proper sense of the term, but it answers admirably the purpose for which it has

been originated, viz.: it pledges a temporary support to those who are left to struggle alone in life by the death of the one upon whom they are dependent.

I have endeavored in the following pages to formulate a system of Tontine Investment that can be utilized to great advantage under the Monthly Redemption Bond System, basing my calculations upon the same rates as those existing now in Life Insurance, but taking into consideration the essential features of a Tontine policy, mentioned above.

RATES.

The annual premiums in Life Insurance, are fixed for every age, according to the American or other Table of Mortality. They range, for $1000, between $18.60 at the age of 21 and $141.70 at the age of 70; the average being about $54 per thousand, or 5.4% annually of face value of policy, which would be equivalent approximately to $\frac{1}{2}\%$ of face value monthly—the exact figures being $\frac{45}{100}\%$.

Under this system, therefore, $\frac{1}{2}\%$ of face value of bond constitutes the monthly rate.

Let f=face value; m=the number of months; mp=monthly payment or rate; P=totality of payments. We have according to the above statement:

(1) $\qquad$ m p$=\frac{1}{2}\%\times$f $\qquad$ or $=.005\times$f
(2) $\qquad$ P$=\frac{1}{2}\%\times$f$\times$m

The rate on $1000 bond $=.005\times1000=$5.$

PERCENTAGE OF PROFIT.

This percentage in Life Insurance system is at its maximum in the first year, and at its minimum in the last year of any policy. It ranges from 100 or less to the thousands; the general average varies between 180 and 200. Under the Equitable system, such order is reversed, so that the maximum of profit is obtained in the remotest, and the minimum in the earliest, period of a bond. In other words, the longer a bond will run and the more payments are made therefor, the greater profit it will accrue.

If we were to distribute the general average of 180 or 200 per cent. among the policy holders during a period of—say 26 years*, in such a way that the long livers get the most, and the short livers get the least profit, graduating the increase uniformly according to a fixed rule, we would, then, be consistent with the principles of investment as applied to the working of the Tontine element. The benefits, in such plan, would be regulated in their distribution and division on a percentage basis, as are all profits from other investment, with the exception of a higher percentage. It is rather immaterial what the starting point should be in this connection, as long as the profit grows greater according to the outlay and age of policy. Let us assume that 100 per cent. is the starting point of gradation, and 6 per cent. an annual addition. The percentage of profit in the first year would be 106. This would mean that, should one die in the first year of his policy, or after

* 26 years is the average of "expectation of life" from the age of 25 to 60 inclusive.

one annual premium had been paid, the amount his heirs are to receive, should be the premium paid, and 106 per cent. of profit. Accordingly, the percentage of profit would be—

In the 2nd year .... $100+(6\times2)$
"   " 3rd   "   .... $100+(6\times3)$
"   " 4th   "   .... $100+(6\times4)$
"   " n   "   .... $100+(6\times n)$

If the average of such series is taken for the 26 years—the average of expectation of life—it would amount approximately to 180—the same as the average of profit developed by the present system of insurance.

Such suggestion is plausible when Life Insurance is treated as an investment rather than a benevolent or fraternal society.

In applying the same plan to the distribution of profits in a legitimate " Bond Enterprise," we simply divide the 6 per cent.—the annual addition—into 12 parts of $\frac{1}{2}$ each for every month. Thus the percentage of profit would be—

In the 1st month .... $100+(\frac{1}{2}\times1)$
"   " 2nd   "   .... $100+(\frac{1}{2}\times2)$
"   " 3rd   "   .... $100+(\frac{1}{2}\times3)$
"   " 4th   "   .... $100+(\frac{1}{2}\times4)$
(3) "   " m   "   .... $100+(\frac{1}{2}\times m)$

and so on indefinitely.

If the average of the above monthly series is taken for 312 months (26 years), it would not exceed 180, the same as the general average developed by the operation of the present insurance system. In fact, it would be the same average as that of the preceding yearly series.

REDEMPTION VALUE.

Knowing from formula (2) the monthly payments, and having regulated above, the percentage of profit in each month, within the limits of the general average that is developed in Life Insurance, we can arrive easily enough at a redemption value of a bond for any month. For example: Wanted the redemption value of a bond, in the second month, face value, 1000.

Formula (1)   m p$=.005 \times$ f, or $.005 \times 1000 = 5$
      (2)   P$=$m$\times$m p, or $2 \times 5 = 10$

10 is the total payments in the second month. Its percentage of profit in 2nd month, according to the above monthly series, is $100 + (\frac{1}{2} \times 2)$ or 101. Therefore take 101% of 10, viz., 10.10, which is the profit over the outlay or principal. Both these elements, i. e., 10.10—the net profit—and 10—the principal— or 20.10, constitute the redemption value for that month.

It will be noticed that 20.10 is equal to 2% of 1000, or 20; plus 1% of 10—the total payments— viz., .10; total, 20.10.

Another example: Wanted redemption value of 1000 in the 15th month. According to Formula (3) we have the percentage of profit$=100 + (\frac{1}{2} \times 15)$ or 107.5.

According to Formula (2), the total payments are 75. Take 107.5% of 75, viz. 80.625, which is the net profit, add to this amount the total payments— 75; total, 155.625, the redemption value of 1000 in

the 15th month. In other words, the redemption value $= \dfrac{(100 + \frac{1}{2} \times 15) \times 75}{100} + 75 = 155.625$.

Therefore we have the following formula:

(4) Redemption value in $m = \dfrac{(100 + \frac{1}{2} \times m) \times P}{100} + P$

But 155.625 can be deduced in another manner: the number of months is 15; take 15% of face value, which is 1000, viz., 150; add to this amount $7\frac{1}{2}\%$ of total payments, viz., 5.625: total, 155.625 as above.

Therefore we have:

$\dfrac{(100 + \frac{1}{2} \times m) \times P}{100} + P = m\% \times f + \dfrac{m}{2}\% \times P$

Consequently,

Redemption value, or $R = m\% \times f + \dfrac{m}{2}\% \times P$ (5)

Or: $R = m\% \times (f + \dfrac{P}{2})$ (6)

Thus we conclude the following rule (Formula 5): Redemption value of a bond equals *the same per cent. of face value as number of months* (or number of monthly payments) *plus half the said per cent. of total payments.*

Or (Formula 6): Redemption value equals *the same per cent., of face value and half the total payments, as the number of months.*

According to the above rule, the redemption value increases in a cumulative manner. The part of formula that creates the cumulative element is $\dfrac{m}{2}\% \times P$; for, as P is increasing every month, therefore $\dfrac{m}{2}\% \times P$ increases also monthly, but cu-

mulatively, *i.e.*, the increase in one month is greater than that in the preceding one, by a fixed ratio.

A new feature can be introduced into Life Insurance by adopting this system to regulate "redemption values" for life policies, which (redemption values) may be paid at death instead of face values. Such system would dispense with the medical examination, which bars a great many from entertaining the privilege of having their life insured even for a small sum. Under such system, if the consumptive desires to have his life insured he could do so, but by his early death, his policy would not accrue greater profit than what is determined by the formula $100+(\frac{1}{2}\times m)$, which is the average of profit. In fact, the policies that terminate soon, would cost proportionately more than those that terminate in a longer period, for the profits on such would be proportionately much greater.

The experience of Life Insurance in the last forty years would, as it is shown in the previous pages, justify the adoption of such views whereby the policy holders or their heirs could know in any period the amount due to them on their policies when terminated.

The table on the following page is computed from the above formulæ for $1000. The redemption value reaches the face value of bond in the 83rd month, and keeps increasing monthly and indefinitely.

Table of Monthly Redemption Values.

For $1000.

Months m	Total Payments $m \times \frac{1}{2} \times f$ or P.	Redemption Value $m \times f$ + $m/2 \times P$	$(100 + \frac{1}{2} \times m)$ P. C. of Profit	Months m	Total Payments $m \times \frac{1}{2} \times f$ or P.	Redemption Value $m \times f$ + $m/2 \times P$	$(100 + \frac{1}{2} \times m)$ P. C. of Profit
1	$5	$10.025	100 5	34	$170	$368.90	117
2	10	20.10	101	35	175	380.625	117.5
3	15	30.225	101.5	36	180	392.40	118
4	20	40.40	102	37	185	404.225	118.5
5	25	50.625	102.5	38	190	416.10	119
6	30	60.90	103	39	195	428.025	119.5
7	35	71.225	103.5	40	200	440.00	120
8	40	81 60	104	41	205	452.025	120.5
9	45	92.025	104.5	42	210	464.10	121
10	50	102 50	105	43	215	476.225	121.5
11	55	113 025	105.5	44	220	488 40	122
12	60	123.60	106	45	225	500.625	122.5
13	65	134.225	106.5	46	230	512 90	123
14	70	144 90	107	47	235	525.225	123.5
15	75	155.625	107.5	48	240	537 60	124
16	80	166 40	108	49	245	550.025	124 5
17	85	177.225	108.5	50	250	562 50	125
18	90	188 10	109	51	255	575.025	125.5
19	95	199.025	109 5	52	260	587.60	126
20	100	210 00	110	53	265	600.225	126.5
21	105	221.025	110 5	54	270	612.90	127
22	110	232.10	111	55	275	625 625	127.5
23	115	243.225	111.5	56	280	638.40	128
24	120	254 40	112	57	285	651.225	128.5
25	125	265.625	112 5	58	290	664.10	129
26	130	276.90	113	59	295	677.025	129.5
27	135	288.225	113 5	60	300	690.00	130
28	140	299.60	114	61	305	703 025	130.5
29	145	311.025	114.5	62	310	716.10	131
30	150	322.50	115	63	315	729.225	131.5
31	155	334.025	115.5	64	320	742.40	132
32	160	345 60	116	65	325	755.625	132.5
33	165	357.225	116.5	66	330	768 90	133

67	$335	$782 225	133.5	94	$470	$1160.90	147
68	340	795.60	134	95	475	1175.625	147.5
69	345	809.025	134.5	96	480	1190.40	148
70	350	822.50	135	97	485	1205.225	148.5
71	355	836.025	135.5	98	490	1220.10	149
72	360	849.60	136	99	495	1235.025	149.5
73	365	863.225	136.5	100	500	1250.00	150
74	370	876.90	137	101	505	1265.025	150.5
75	375	890 625	137.5	102	510	1280 10	151
76	380	904.90	138	103	515	1295.225	151.5
77	385	918.225	138.5	104	520	1310.40	152
78	390	932.10	139	105	525	1325.625	152.5
79	395	946.025	139.5	106	530	1340.90	153
80	400	960 00	140	107	535	1356.225	153.5
81	405	974 025	140.5	108	540	1371.60	154
82	410	988 10	141	109	545	1387.025	154.5
83	415	1002.225	141.5	110	550	1402 50	155
84	420	1016.40	142	111	555	1418.025	155.5
85	425	1030.625	142.5	112	560	1433.60	156
86	430	1044.90	143	113	565	1449.225	156.5
87	435	1059.225	143.5	114	570	1464.90	157
88	440	1073.60	144	115	575	1480.625	157.5
89	445	1088.025	144.5	116	580	1496.40	158
90	450	1102.50	145	117	585	1512.225	158.5
91	455	1117.025	145.5	118	590	1529.10	159
92	460	1131 60	146	119	595	1544.025	159.5
93	465	1146.225	146.5	120	600	1560.00	160

Etc., Etc.

It will be noticed in the preceding table, that the percentage of profit is always one hundred plus half the number of months, which is the same as $100 + (\frac{1}{2} \times m)$.

It is thus shown that the Equity system of Cumulative Tontine Investment is based upon a demonstrable mathematical principle, which entirely divorces it from arbitrary adoption and chance element of speculation, which is so generally applied to the many bond enterprises heretofore in existence.

PART II.

MODE OF TERMINATION OF POLICIES AND BONDS.

The liabilities in every system of insurance depend always upon certain contingencies. In Life Insurance that contingency is death; in Fire Insurance, fire; in Accident Insurance, accident. The termination of policies under the three systems depends upon those contingencies. In the Endowment system, policies terminate at the expiration of a definite period. If that period is not long enough to demonstrate the operation of the Tontine element, failure will be the inevitable result. This is exemplified in the so many *short term orders* that rise into existence for a time, and then vanish, having no more energy and tenacity in them, than a soap-bubble.

In a bond enterprise, according to the idea of the originator, the liabilities exist only when there are funds to meet them, consequently there is no possibility of failure. But the most serious question is, how and when should a certain number of bonds terminate, when there is no contingency either of death, or of fire, or of accident.

The practical principle that governs the operation of all these enterprises is the same. 1000 men insure their lives; 1000 houses are insured against fire; 1000 ships are insured against accident; 1000 bonds issued by a bond company. Let us assume that 10 policies will have to be terminated in each. In

the first class *Death* will pick out the weakest 10
persons, whose physical constitutions are impaired
by natural causes, and sweep them out of existence;
their policies are paid from what the 1000 had paid
in. In the second class *Fire* will burn the 10 houses
which have been subjected to more favorable con-
ditions for fire to operate, than the 990 remaining
houses. In the third, *Accident* caused by unseen
and unexpected forces—yet these forces are guided
by law of nature—destroys the first 10 ships that
are led to the place, and during the time, when
such forces are being developed. In the fourth
class, 10 bonds can be surrendered and paid from
what the 1000 had paid in also. The only question
that arises is, what 10 bonds among the 1000 bonds
issued will have to terminate without working an
inequality to those remaining. The usual way prac-
ticed by bond companies heretofore is to terminate
the ten bonds in numerical order, that is to say, to
redeem or pay off the first ten, numbered in the
series 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 &c. In some in-
stances the numerical order has been deviated from,
and an order of the numbers and their multiple of
five has been adopted. The first ten bonds would
be: 1, 5, 2, 10, 3, 15, 4, 20, 5, 25.

In another instance a system of multiples of 3 and
7 is adopted. Thus the first ten bonds would be:

| 1, then 3, then 7 | 3, then 9, then 21. |
| 2 " 6 " 14 | 4, and so on. |

The multiple of 3 has been used also, and the
first ten numbers would be: 1, 3, 9, 2, 6, 18, then
No. 27, then 4, 12, 36, and so on.

In all these methods there is one *important fact,* viz., that the numbers of bonds to be redeemed first is known beforehand; hence purchased first in preference to all others, whether said bonds are numbered 1, 2, 3, 4, 5, &c.; or 1, 5, 2, 10, 3, 15, &c.; or 1, 3, 7, 2, 6, 14, &c.; or 1, 3, 9, 2, 6, 18, &c. So long as the order of redemption is definitely known, so long the element of preference exists, which is the most serious objection in such enterprises.

But if the termination of bonds is made to depend upon certain contingency, which is as uncontrollable and beyond human prediction as is that of death, fire or accident—yet a physical or mathematical law—then there will be no difference in the mode of termination of policies in the four classes, as the practical principle which underlies them all, is one and the same.

METHODS OF REDEMPTION.

In a series of bonds issued and numbered from 1 upward, one method—by which the bonds to be redeemed depend upon certain contingency—may consist in redeeming every certain number; say every 20th, 40th or 50th bond in force, beginning with number 1, counting to one end of the series and, if necessary, carrying the count at the same rate over to the first end again, and so on, till the number of bonds redeemed exhaust the redemption fund to be distributed. The last bond redeemed in one month will be the starting point for the count in the next month. In this manner, all bonds that

terminate by lapse, redemption or otherwise, would be discarded in the count, and all that are in force would be equidistantly subjected to redemption in an equitable and uniform manner. But such method is not exempt from some objectionable features which may render it cumbersome in practice.

Another method, which is more regular and accurate in operation, as well as mathematical in principle, is a modification of the preceding one.

In order to illustrate its details, let us assume that a registered bond in a general series running from 1 upward consecutively, is not eligible for redemption until it shall have been four months in force.

The modification consists in having the number that designates the interval between the numbers of bonds to be redeemed, equivalent to one per cent. (less fractions) of the number of the highest eligible bond at time of redemption, instead of having an arbitrary number as in the first method. In this way *that number* varies every month if redemptions are made monthly, and always bears a steady ratio to all eligible bonds among which redemption is to be made. It varies in the same ratio as the number of bonds in the general series increases.

To illustrate. Let us assume that the number of the highest eligible bond—the last registered bond that has been four months in force—in making the first redemption, is 850. One per cent. (less fraction) of said number is 8; therefore, bonds are redeemed in this instance 8 numerals apart. Begin-

ning with No. 1,—the beginning of the general series,—we have the following numbers that determine the bonds to be redeemed, viz.:

1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105, &c.

It will be observed that these numbers form a mathematical sub-series,* whose first term is one, common difference 8, and the last term 105.

Assuming that 105 is the number of the last bond redeemed in the first redemption, it then constitutes the starting-point in the next month's redemption, or the first term of the next month's sub series. Suppose we find that the highest eligible bond in this month is 1230; then 12 denotes the numerals apart, or the common difference; the sub-series would be: $105+12=117$, 129, 141, 153, 165, 177, 189, 201, 213, &c.

Again, let us assume that the number of the highest eligible bond in the third month is 1549, and the number of the last bond redeemed in the previous month is 585, then 15—1% (less fraction) of 1540—denotes the numerals apart or the common difference for this month. The numbers of bonds to be redeemed would be: $585+15=600$, 615, 630, 645, 660, 675, 690, &c. Let us also assume that we have to redeem 70 bonds in this month; this means that we have to extend this sub-series into 70 terms. By adding 15, 65 times to 585, we will arrive to 1560, which number exceeds the number of the

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* The word sub-series is used here in distinction of the general series, the first term of which is 1 and its common difference is also 1 : in other words, it is the numbers 1, 2, 3, 4, 5, 6, 7, 8, &c.

highest eligible bond, which is 1549. Therefore the term 1560 could not be considered for redemption, as its number is not eligible; but we return to the lowest number in force—say 2, and form the numbers of the five remaining bonds to be redeemed, thus: $2+15=17, 32, 47, 62$, the last bond redeemed. Hence we establish the following rule, viz. :

The numbers which determine the bonds to be redeemed in a month shall form a mathematical sub-series, whose first term is the number of the last bond redeemed in a previous month, and its common difference is one per cent. (less fractions) of the number of the highest eligible bond in that month. The bonds in force, whose numbers correspond to the terms of such sub-series, are redeemed consecutively, always commencing at the beginning of the sub series. Whenever one term is terminated by redemption, lapse or death of the holder, the next term will be chosen for redemption. Whenever the number of the highest eligible bond is exceeded by any term, the number of the lowest bond in force may constitute the next term of the sub series.

The following tables and example on the next page serve to illustrate the working of this method. The number of bonds issued, the date of issue and number of bonds redeemed in each month are all assumed. Bonds whose numbers correspond to the term of the sub-series are consecutively redeemed, and as many of them redeemed as the redemption fund for the month wholly covers.

EXAMPLE

OF

THE MATHEMATICAL METHOD OF REDEMPTION IN ACTUAL OPERATION.

Date of issue.	Number of the highest eligible bond.	$\frac{1}{2}\%$ (less fraction) of the number of the highest eligible bond or common difference.	Number of bonds redeemed.	TERMS OF THE MONTHLY SUB-SERIES.
				TERMS OF SUB-SERIES FOR THE FIRST REDEMPTION, MADE MAY 1st.
Jan. 31	620	6	89	1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73. 79, 85, 91, 97, 103, 109, 115, 121, 127, 133, 139, 145, 151, 157, 163, 169, 175, 181, 187, 193, 199, 205, 211, 217, 223, 229, 235, 241, 247, 253, 259, 265, 271, 277, 283, 289, 295, 301, 307, 313, 319, 325, 331, 337, 343, 349, 355, 361, 367, 373, 379, 385, 391, 397, 403, 409, 415, 421, 427, 433, 439, 445, 451, 457, 463, 469, 475, 481, 487, 493, 499, 505, 511, 517, 523, 529, last bond redeemed.

				TERMS OF SUB-SERIES FOR THE SECOND REDEMPTION, MADE JUNE 1st.
Feb. 28	1040	10	54	$529 + 10 = 539, 549, 559, 569,$

579, 589, 599. 609, 619, 629, 639, 649, 659. 669, 679, 689, 699, 709, 719, 729. 739, 749, 759, 769, 779. 789. 799, 809, 819, 829, 839, 849, 859, 869, 879, 889, 899, 909. 919, 929, 939, 949, 959, 969. 979, 989, 999, 1009, 1019. 1029, 1039. 1049, this number is not eligible for redemption, as the last number eligible is **1040.** Therefore the number of the lowest bond in force constitutes the next term to **1039,** viz., Nos. 2, 12. 22, 32, 42, 52, the last bond redeemed.

TERMS OF SUB-SERIES FOR THE THIRD REDEMPTION, MADE JULY 1st.

Mar. 31	1406	14	46	$52 + 14 = 66, 80, 94, 108, 122,$

136, 150, 164, 178, 192, 206, 220, 234, 248, 262, 276, 290, 304, 318, 332, 346, 360, 374. 388, 402, 416, 430. 444, 458, 472, 486, 500, 514, 528, 542, 556, 570, 584, 598, 612, 626, 640, 654, 668, 682, 696, 710, 724, last bond redeemed.

TERMS OF SUB-SERIES FOR THE FOURTH REDEMPTION, MADE AUG. 1st.

Apr. 30	1515	15	15	$724 + 15 = 739. 754, 769, 784,$

799, 814, 829. 844, 859. 874, 889. 904, 919. 934, 949. 964, 979. 994, 1009. 1024, 1039. 1054, 1069, 1084, 1099, 1114. last bond reedeemed.

May 31	1870
June 30	2030
July 31	2260
Aug 31	2850

NOTE.—The underlined numbers indicate the bonds that have been previously redeemed or terminated.

We conclude from the preceding table and examples the following :

Bonds are subject to redemption when their numbers correspond to the terms of a mathematical sub-series having No. 1 in the first redemption, and the the number of the last bond redeemed in subsequent redemptions, for its *first term*, and 1% (less fraction) of the number of the highest eligible bond for its *common difference*. But each month's redemption commences with the first term of such sub-series, redeeming consecutively as many corresponding bonds in force as the redemption fund for that month will wholly cover.

It must be remembered that, when any term of such sub-series, as formed by the preceding rule, exceeds the number of the highest eligible bond, that term is annulled from the said sub-series, and substituted by the number of the lowest bond in force from which proceeds the regular count for the formation of more terms if needed. In such instance the sub-series seems to be formed of two divisions, as is the case in the sub-series for the second redemption, where the first division begins with No. 529, and ends with No. 1039 ; the second division begins with No. 12, and ends with No. 52, but both divisions have the same common difference, being for the same month.

THE BASIS OF THIS METHOD

is the fundamental law of the mathematical series. The formula of such series can be used to insure correctness in the count.

Let a=first term; l=last term; d=common difference; n=numbers of terms.

We have therefore:

$$l = (n - 1) \times d + a$$

Or the last term equals the common difference multiplied by the number of terms less one, plus the first term.

In the sub-series, for every month's redemption, the first term (a) is positively known, being No. 1 in the first redemption, and the number of the last bond redeemed in the subsequent redemptions. So also the common difference (d) is known, being always 1% (less fraction) of the number of the highest eligible bond. But the number of terms (n) depends absolutely upon the amount of the redemption fund to be distributed in each month, and therefore it is variable.

The commencement of all the sub-series is the *known quantity* 1, and from this quantity we form the unknown terms according to the rules laid down previously.

Referring to the previous example, page 22, if the redemption fund in the first month were enough to redeem Nos. 1 and 7, then the number of terms or n would equal 2; or if it were enough to redeem 1, 7, 13, 19, then n=4, and so on. The terms, therefore, in each sub-series are formed gradually by our redeeming as many bonds as the redemption fund covers.

Let $\times$ = the next term or number to be determined for redemption, we have then:

$$\times = (n \times d) + a$$

In the above example, where we had Nos. 1, 7, 13 and 19 redeemed, if we want to know the next number to be determined for redemption, we have:

Next Number		Number of Terms	Common Difference		First Term		
$\times$	=	(n	$\times$	d	) +	a	= 25
		4		6		1	

or $\times = 25$, the next number to be determined.

The number of terms, or n, are now 5, viz.: 1, 7, 13, 19, 25. The next term or $\times = (5 \times 6) + 1 = 31$, and so on.

Forming a sub-series for every month's redemption, and making the starting-point of redemption the *known* quantity 1, we have the following unchangeable and important formula, viz.:

$$\times = (n \times d) + a$$

This formula creates the following rule:

The next number to be determined for redemption equals the number of terms already formed multiplied by the common difference, plus the first term of the monthly sub-series.

Every number that is redeemed in the previous example conforms exactly with this rule. It must be remembered, however, that whenever a sub-series is composed of two divisions as in the second sub-series, this rule works in each division separately; that is to say, each division is considered as

separate sub-series, but both have the same common difference.

SUMMARY

OF

REDEMPTION METHOD.

Referring to the general series above, we may say briefly, that this method consists in redeeming first No. 1 of the general series; then the bond, which is as many numbers removed from No. 1, as the numeral which expresses 1% (less fraction) of the number of the highest eligible bond at time of redemption; then the one which is as many numerals removed from the last bond redeemed, as that percentage, and so on, counting from the last bond redeemed each time. When such indicated number is terminated, the count extends therefrom to the next number indicated in the same manner; such eligible bonds being redeemed as said count falls upon. When said count exceeds the highest eligible bond, the lowest in force shall be the next redeemed. Each successive month's redemption will proceed from the last bond redeemed the previous month.

DEDUCTION.

It will be observed that the number of the last bond redeemed in each month plays a very important part in this method; forming the last term of one sub-series and the first term of another. The determination of this number depends:

1st. *Upon the amount of the monthly redemption fund.* In the sub series for the first redemption, if the amount of the redemption fund were only enough to reach in redemption No. 517, instead of No. 259, then all the terms of the subsequent sub-series would have been different.

2nd. *Upon the lapse element.* In the sub series for the second redemption, the terms 659, 729, 909 correspond to lapsed bonds. Had these bonds been in force at time of redemption, they would have been redeemed, and the last bond redeemed in that month would have been 22 instead of 52. Consequently all subsequent bonds to be redeemed would have been different.

3rd. *Upon the numbers of redeemed or terminated bonds.* In the sub-series for the fourth redemption it is evident enough that, if the 12 underlined numbers had not been previously redeemed, the 15 bonds redeemed in that month would have ended with No. 949 instead of No. 1114.

4th. *Upon, of course, the common difference of the sub-series,* which is always equivalent to 1% (less fractions) of the number of the highest eligible bond, which number varies, and depends entirely upon the number of bonds sold in each month.

CONCLUSION.

It is evidently shown in the foregoing elucidation that the Equity System of Cumulative Tontine Investment is based upon purely mathematical basis, which basis underlies the practical principle of Life Insurance.

The termination of policies on bonds under this system depends wholly upon contingencies which are beyond the control of any management; hence the element of preference, as well as the fraudulent practices developed by the abuse of such principle, are entirely obviated.